

Apportionment Paradoxes

Lecture 25
Section 4.6

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- 1 The Quota Rule
- 2 Some History
- 3 Apportionment Paradoxes
 - The Alabama Paradox
 - The Population Paradox
 - The New-States Paradox
- 4 The Current Congress
- 5 Assignment

Outline

- 1 The Quota Rule
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The Quota Rule

Definition (The Quota Rule)

The **quota rule** says that the number of representatives apportioned to each state should be at least that state's lower quota, but not more than that state's upper quota.

- As we have already seen, Hamilton's method is the only method that is *guaranteed* to satisfy the quota rule.
- Is the quota rule fair?

The Quota Rule

Example (Jefferson's Method and the Quota Rule)

- Let 5 states have populations 1, 2, 3, 4, and 11 million people.
- Apportion 28 seats by Jefferson's method.
- Apportion 90 seats by Adams's method.

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History

- Jefferson's method was used in every apportionment from 1790 through 1830.
- In 1840, Congress adopted Webster's method.
- From 1850 through 1900, Hamilton's and Webster's methods were used. In each case, they produced the same result.
- From 1910 through 1930, Webster's method was used.
- The size of the House was steadily increased until 1929 when it was fixed at 435 seats.
- From 1940 to today, the Huntington-Hill method has been used.

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The Alabama Paradox

- After the 1880 census, Congress had to decide how many House seats there would be, and then apportion them.
- If they created 299 House seats, then Alabama would get 8 seats.
- But if they created 300 House seats, then Alabama would get only 7 seats.

The Alabama Paradox

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- If they created 299 House seats, then Alabama would get 8 seats.
- But if they created 300 House seats, then Alabama would get only 7 seats.
- How can that be?

The Alabama Paradox

Definition (The Alabama Paradox)

The **Alabama paradox** occurs when a state is apportioned *fewer* seats when one new seat is *added*, even though none of the populations changed.

The Alabama Paradox

Example (Stolen from Wikipedia)

- Let states A , B , and C have populations of 2.1, 6.2, and 6.3 million.
- Compute the apportionment, under Hamilton's method, if there are 10 seats total.
- Add one seat for a total of 11 and reapportion.

The Alabama Paradox

Example (Stolen from Wikipedia)

- Let states A , B , and C have populations of 2.1, 6.2, and 6.3 million.
- Compute the apportionment, under Hamilton's method, if there are 10 seats total.
- Add one seat for a total of 11 and reapportion.
- Does the same thing happen under the other methods?

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The Population Paradox

- From 1890 to 1900, Virginia's population grew much faster than Maine's population.
- However, when the seats were reapportioned, Virginia lost a seat and Maine gained a seat.

The Population Paradox

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- How can that be?

The Population Paradox

Definition (The Population Paradox)

The **population paradox** occurs when one state loses a seat and another state gains a seat, even though the first state's population increased more than the second state's population.

The Population Paradox

Example (The Population Paradox)

- Let states A , B , and C have populations 13, 12, and 112 million, respectively, with 25 seats to be apportioned.
- Calculate the number of seats apportioned, using Hamilton's method.
- Add 1 million to A 's population and 2 million to C 's population and recalculate the apportionment.

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The New-States Paradox

- In 1907, Oklahoma was admitted to the union.
- There were 386 seats in the House.
- Based on Oklahoma's population, it deserved to get 5 seats, so the total was raised to 391 seats.
- When the seats were reapportioned, Maine gained a seat and New York lost a seat.

The New-States Paradox

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- There were 386 seats in the House.
- Based on Oklahoma's population, it deserved to get 5 seats, so the total was raised to 391 seats.
- When the seats were reapportioned, Maine gained a seat and New York lost a seat.
- How can that be?

The New-States Paradox

Definition (The New-States Paradox)

The **new-states paradox** occurs when a new state is added and the number of seats is increased by the new state's fair share, yet the number of seats apportioned to the other states changes.

The New-States Paradox

Example (The New-States Paradox)

- Let states A and B have populations 52 and 134 million, respectively, with 16 seats to be apportioned.
- Calculate the number of seats apportioned, using Hamilton's method.
- Add a new state C with a population of 39 million and recalculate the apportionment.

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The Current Congress

The Current Congress

- Calculate the apportionment of the 115th Congress (the current Congress) using the Huntington-Hill method.
- Recalculate it, using the other four methods: Hamilton's, Jefferson's, Adams's, and Webster's.
- Are there any differences?

Comparisons with the Current Congress

Example (Comparisons with the Current Congress)

| State | Ham | Jeff | Adams | Web | Hill |
|-------|-----|------|-------|-----|------|
| CA | 53 | 55 | 50 | 53 | 53 |
| DE | 1 | 1 | 2 | 1 | 1 |
| FL | 27 | 28 | 26 | 27 | 27 |
| GA | 14 | 14 | 13 | 14 | 14 |
| ID | 2 | 2 | 3 | 2 | 2 |
| IL | 18 | 19 | 18 | 18 | 18 |
| IA | 4 | 4 | 5 | 4 | 4 |
| LA | 6 | 6 | 7 | 6 | 6 |
| ME | 2 | 1 | 2 | 2 | 2 |
| MN | 8 | 7 | 8 | 8 | 8 |
| MO | 8 | 8 | 9 | 8 | 8 |
| MT | 1 | 1 | 2 | 1 | 1 |
| NE | 3 | 2 | 3 | 3 | 3 |
| NH | 2 | 1 | 2 | 2 | 2 |

Comparisons with the Current Congress

Example (Comparisons with the Current Congress)

| State | Ham | Jeff | Adams | Web | Hill |
|-------|-----|------|-------|-----|------|
| NJ | 12 | 13 | 12 | 12 | 12 |
| NY | 27 | 28 | 26 | 27 | 27 |
| NC | 13 | 14 | 13 | 14 | 13 |
| OH | 16 | 17 | 16 | 16 | 16 |
| OK | 5 | 5 | 6 | 5 | 5 |
| OR | 5 | 5 | 6 | 5 | 5 |
| RI | 2 | 1 | 2 | 1 | 2 |
| SC | 7 | 6 | 7 | 7 | 7 |
| SD | 1 | 1 | 2 | 1 | 1 |
| TX | 36 | 37 | 34 | 36 | 36 |
| VT | 1 | 0 | 1 | 1 | 1 |
| WA | 10 | 10 | 9 | 10 | 10 |
| WV | 3 | 2 | 3 | 3 | 3 |
| WY | 1 | 0 | 1 | 1 | 1 |

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Assignment

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- Ch. 4: Exercises 51, 52, 55, 56, 58, 61, 62.